Solve the system of differential equation $\begin{cases} x' = 4x - 2y \\ y' = x + y \end{cases}$, $x(0) = 3$, $y(0) = 2$. [104 交大土木丁 4]

[解]令$D = \dfrac{d}{dt}$, 原式$\Rightarrow \begin{cases} (D-4)x + 2y = 0 \cdots\cdots\cdots\cdots\cdots\cdots \text{(i)} \\ -x + (D-1)y = 0 \cdots\cdots\cdots\cdots\cdots\cdots \text{(ii)} \end{cases}$

由(ii) $x = (D-1)y \cdots\cdots\cdots \text{(iii)}$，代入(i)

$(D-4)(D-1)y + 2y = 0 \Rightarrow (D^2 - 5D + 6)y = 0 \Rightarrow (D-2)(D-3)y = 0$

$y = C_1 e^{2t} + C_2 e^{3t}$，代入(iii)

$x = (D-1)y = (2C_1 e^{2t} + 3C_2 e^{3t}) - (C_1 e^{2t} + C_2 e^{3t}) = C_1 e^{2t} + 2C_2 e^{3t}$

$\begin{cases} x(0) = 3 \Rightarrow C_1 + 2C_2 = 3 \\ y(0) = 2 \Rightarrow C_1 + C_2 = 2 \end{cases} \Rightarrow C_1 = 1, C_2 = 1$

解為 $x = e^{2t} + 2e^{3t}$, $y = C_1 e^{2t} + C_2 e^{3t}$