Please find the ODE $(x+1)\dfrac{dy}{dx}+y=\ln x$, $y(1)=10$. [104 中興精密 1(b)]

[解]原式 $\Rightarrow y' + \dfrac{y}{x+1} = \dfrac{\ln x}{x+1}$

$F = e^{\int \frac{1}{x+1}dx} = e^{\ln(x+1)} = x+1$

$y = \dfrac{1}{F}(\int F \cdot \dfrac{\ln x}{x+1}dx + C) = \dfrac{1}{x+1}(\int \ln x\,dx + C) = \dfrac{1}{x+1}(x\ln x - x + C)$

$y(1) = 10 \Rightarrow \dfrac{1}{2}(0-1+C) = 10 \Rightarrow C = 21$

$y = \dfrac{1}{x+1}(x\ln x - x + 21)$