

Solve the differential equation  $(x+3)^2y'' - 8(x+3)y' + 14y = 0$ . [104 中興機械 1]

[解] 令  $x+3 = z \Rightarrow \frac{d}{dx} = \frac{d}{dz}$

原式  $\Rightarrow z^2 \frac{d^2y}{dz^2} - 8z \frac{dy}{dz} + 14y = 0 \dots\dots\dots$  (i)

令  $z = e^t \Rightarrow t = \ln z = \ln(x+3)$

(i) 式  $\Rightarrow \frac{d^2y}{dt^2} - 9 \frac{dy}{dt} + 14y = 0 \dots\dots\dots$  (ii)

(ii) 式的特徵方程式  $\lambda^2 - 9\lambda + 14 = 0 \Rightarrow (\lambda - 2)(\lambda - 7) = 0 \Rightarrow \lambda = 2, 7$

$y = C_1e^{2t} + C_2e^{7t} = C_1(x+3)^2 + C_2(x+3)^7$

