

Solve  $(x^2 + y^2 + x)dx + xydy = 0$ . Whether the equation is exact or not. Find an integrating factor and the solution. [104 中正化工 2]

$$[\text{解}] M = x^2 + y^2 + x, N = xy \Rightarrow \frac{\partial M}{\partial y} = 2y, \frac{\partial N}{\partial x} = y$$

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{2y - y}{xy} = \frac{1}{x}$$

$$\mu = e^{\int \frac{1}{x} dx} = e^{\ln x} = x$$

$$u = \int_x \mu M dx + f(y) = \int_x x(x^2 + y^2 + x) dx + f(y)$$

$$= \frac{x^4}{4} + \frac{x^2 y^2}{2} + \frac{x^3}{3} + f(y)$$

$$\frac{\partial u}{\partial y} = \mu N \Rightarrow x^2 y + f'(y) = x(xy) \Rightarrow f'(y) = 0 \Rightarrow f(y) = 0$$

$$\text{解為 } \frac{x^4}{4} + \frac{x^2 y^2}{2} + \frac{x^3}{3} = k \Rightarrow 3x^4 + 6x^2 y^2 + 4x^3 = C$$