Find general solution $y(x)$ for the ordinary differential equation $y' + y - x\sqrt{y} = 0$. [104 中山光電 3(a)]

[解]原式 $\Rightarrow y' + y = x\sqrt{y} \Rightarrow y^{-\frac{1}{2}}y' + y^{\frac{1}{2}} = x \cdots\cdots\cdots$ (i)

令 $u = y^{\frac{1}{2}} \Rightarrow u' = \frac{1}{2}y^{-\frac{1}{2}}y'$

$$\text{(i)} \Rightarrow 2u' + u = x \Rightarrow u' + \frac{1}{2}u = \frac{1}{2}x$$

$$F = e^{\int \frac{1}{2}dx} = e^{\frac{1}{2}x}$$

$$u = \frac{1}{e^{\frac{1}{2}x}}[\int e^{\frac{1}{2}x} \cdot \frac{1}{2}xdx + C] = e^{-\frac{1}{2}x}[(xe^{\frac{1}{2}x} - \int e^{\frac{1}{2}x}dx) + C] = e^{-\frac{1}{2}x}[(xe^{\frac{1}{2}x} - 2e^{\frac{1}{2}x}) + C]$$

$$= x - 2 + Ce^{-\frac{1}{2}x}$$

$$y = u^2 = (x - 2 + Ce^{-\frac{1}{2}x})^2$$