$(e^{2y}-y\cos xy)dx+(2xe^{2y}-x\cos xy+2y)dy=0$. [104 中原機械甲 2]

[解] $M=e^{2y}-y\cos xy \Rightarrow \dfrac{\partial M}{\partial y}=2e^{2y}-\cos xy-y(-x\sin xy)=2e^{2y}-\cos xy+xyx\sin xy$

$N=2xe^{2y}-x\cos xy+2y \Rightarrow \dfrac{\partial N}{\partial x}=2e^{2y}-\cos xy-x(-y\sin xy)=2e^{2y}-\cos xy+xyx\sin xy$

$\dfrac{\partial M}{\partial x}=\dfrac{\partial N}{\partial y}$ ⇒原式為正合微分方程式

$u=\int_x M\,dx+k(y)=\int_x (e^{2y}-y\cos xy)\,dx+k(y)=xe^{2y}-\sin xy+k(y)$

$\dfrac{\partial u}{\partial y}=N \Rightarrow 2xe^{2y}-x\cos xy+k'(y)=2xe^{2y}-x\cos xy+2y \Rightarrow k'(y)=2y \Rightarrow k'(y)=y^2$

解為 $xe^{2y}-\sin xy+y^2=C$