

$(2xy + 2\sin 2x)dx + x^2dy = 0$  為一階常微分方程式，且  $x = \pi/4$  時， $y = -16/\pi^2$ ，求解  $y(x)$ 。[103 高  
第一營建 2]

$$[\text{解}] M = 2xy + 2\sin 2x \Rightarrow \frac{\partial M}{\partial y} = 2x$$

$$N = x^2 \Rightarrow \frac{\partial N}{\partial x} = 2x$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \Rightarrow \text{原式為正合微分方程式}$$

$$u = \int_x M dx + f(y) = \int_x (2xy + 2\sin 2x) dx + f(y) = x^2y - \cos 2x + f(y)$$

$$\frac{\partial u}{\partial y} = N \Rightarrow x^2 + f'(y) = 2x \Rightarrow f'(y) = 0 \Rightarrow f(y) = 0$$

$$\text{解為 } u = C \Rightarrow x^2y - \cos 2x = C$$