

$$y'' + 2y(y')^3 = 0. \text{ [103 清大生醫甲 5(d)]}$$

$$\text{[解] 令 } p = y' \Rightarrow y'' = \frac{dy'}{dx} = \frac{dy'}{dy} \frac{dy}{dx} = p \frac{dp}{dy}$$

$$\text{原式} \Rightarrow p \frac{dp}{dy} + 2yp^3 = 0 \Rightarrow \frac{dp}{p^2} + 2ydy = 0 \Rightarrow \int \frac{dp}{p^2} + \int 2ydy = C_1 \Rightarrow -\frac{1}{p} + y^2 = C_1$$

$$-\frac{dx}{dy} + y^2 = C_1 \Rightarrow -dx + (y^2 - C_1)dy = 0 \Rightarrow -\int dx + \int (y^2 - C_1)dy = C_2$$

$$-x + \left(\frac{y^3}{3} - C_1 y\right) = C_2$$

南臺科技大學

Southern Taiwan University of Science and Technology