Solve the differential equation $\frac{3y^2-t^2}{y^5}\frac{dy}{dt}+\frac{t}{2y^4}=0, y(1)=1$. [103 清大生醫甲 5(b)]

[解]原式 $\Rightarrow ytdt+(6y^2-2t^2)dy=0$

$M=yt, N=6y^2-2t^2 \Rightarrow \frac{\partial M}{\partial y}=t, \frac{\partial N}{\partial t}=-4t$

$\frac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{-M}=\frac{t-(-4t)}{-yt}=-\frac{5}{y}$

$\mu=e^{\int -\frac{5}{y}dt}=y^{-5}$

$u=\int_t \mu M dt+f(y)=\int_t y^{-5}(yt)dt+f(y)=\frac{y^{-4}t^2}{2}+f(y)$

$\frac{\partial u}{\partial y}=\mu N \Rightarrow -2y^{-5}t^2+f'(y)=y^{-5}(6y^2-2t^2) \Rightarrow f'(y)=6y^{-3} \Rightarrow f(y)=-3y^{-2}$

解為 $\frac{y^{-4}t^2}{2}-3y^{-2}=C$