

The problem is exact, please find its solution $y' = -\frac{2xy^3 + 2}{3x^2y^2 + 8e^{4y}}$. [101 東海電機 6(b)]

[解]原式 $\Rightarrow (2xy^3 + 2)dx + (3x^2y^2 + 8e^{4y})dy = 0$

$$M = 2xy^3 + 2 \Rightarrow \frac{\partial M}{\partial y} = 6xy^2$$

$$N = 3x^2y^2 + 8e^{4y} \Rightarrow \frac{\partial N}{\partial x} = 6xy^2$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \Rightarrow \text{原式為正合}$$

$$u = \int_x M dx + f(y) = \int_x (2xy^3 + 2) dx + f(y) = x^2y^3 + 2x + f(y)$$

$$\frac{\partial u}{\partial y} = N \Rightarrow 3x^2y^2 + f'(y) = 3x^2y^2 + 8e^{4y} \Rightarrow f'(y) = 8e^{4y} \Rightarrow f(y) = 2e^{4y}$$

解為 $x^2y^3 + 2x + 2e^{4y} = C$