

Solve the ordinary differential equation $y' - xy^2 - (1 - 2x)y - x = -1$. [101 台科大自控 1(1)]

[解]原式 $\Rightarrow y' - x(y^2 - 2y + 1) = y - 1 \Rightarrow y' - x(y - 1)^2 = y - 1 \Rightarrow y' - (y - 1) = x(y - 1)^2$
 $(y - 1)^{-2}y' - (y - 1)^{-1} = x \cdots \cdots \cdots \text{(i)}$ 令 $u = (y - 1)^{-1} \Rightarrow u' = -(y - 1)^{-2}y'$ ，代入(i)式
 $-u' - u = x \Rightarrow u' + u = -x$

$$F = e^{\int dx} = e^x$$

$$u = \frac{1}{F} \left[\int F \cdot (-x) dx + C \right] = \frac{1}{e^x} \left[\int e^x \cdot (-x) dx + C \right] = e^{-x} \left[-(xe^x - \int e^x dx) + C \right]$$
$$= e^{-x} \left[-(xe^x - e^x) + C \right] = e^{-x} \left[(-x + 1)e^x + C \right] = -x + 1 + Ce^{-x}$$

$$(y - 1)^{-1} = -x + 1 + Ce^{-x} \Rightarrow \frac{1}{y - 1} = -x + 1 + Ce^{-x}$$

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