Find the most general solution to the ODE $3\frac{dy}{dx} - y = xe^{x/3}$. What is the unique solution satisfying $y(0) = 1$. [100 清大動機 3]

[解]原式 $\Rightarrow y' - \frac{1}{3}y = \frac{1}{3}xe^{x/3}$

$$F = e^{\int -\frac{1}{3}dx} = e^{-x/3}$$

$$y = \frac{1}{F}(\int F \cdot 0dx + C) = e^{x/3}(\int e^{-x/3} \cdot \frac{1}{3}xe^{x/3}dx + C) = e^{x/3}(\frac{1}{6}x^2 + C)$$

$$y(0) = 1 \Rightarrow C = 1 \Rightarrow y = e^{x/3}(\frac{1}{6}x^2 + 1)$$

南臺科技大學
Southern Taiwan University of Science and Technology