

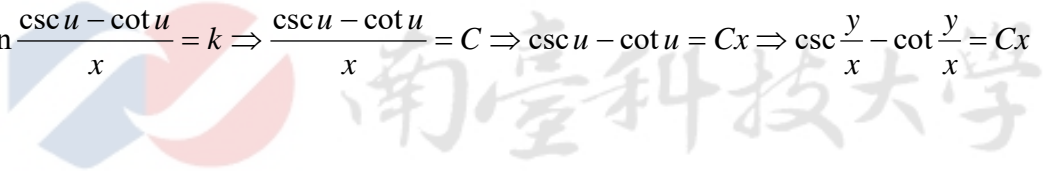
Solve the differential equation $xy' - y - x\sin\left(\frac{y}{x}\right) = 0$. [100 嘉大土木 2(1)]

[解] 令 $y = ux \Rightarrow y' = xu' + u$ ，原式為

$$x(xu' + u) - ux - x\sin u = 0 \Rightarrow (xu' + u) - u - \sin u = 0 \Rightarrow xu' - \sin u = 0$$

$$x \frac{du}{dx} - \sin u = 0 \Rightarrow \frac{du}{\sin u} - \frac{dx}{x} = 0 \Rightarrow \int \frac{du}{\sin u} - \int \frac{dx}{x} = k \Rightarrow \ln(\csc u - \cot u) - \ln x = k$$

$$\ln \frac{\csc u - \cot u}{x} = k \Rightarrow \frac{\csc u - \cot u}{x} = C \Rightarrow \csc u - \cot u = Cx \Rightarrow \csc \frac{y}{x} - \cot \frac{y}{x} = Cx$$



Southern Taiwan University of Science and Technology