

Find the Fourier expansion of the function $f(x) = x^2; (-\pi < x < \pi)$. [98 高應大機械 7]

[解] $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx),$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} x^2 dx = \frac{2\pi^2}{3},$$

$$\begin{aligned} a_n &= \frac{1}{\pi} \int_{-\pi}^{\pi} x^2 \cos nxdx = \frac{2}{n\pi} (x^2 \sin nx \Big|_0^{\pi} - \int_0^{\pi} 2x \sin nxdx) \\ &= \frac{4}{n^2\pi} (x \cos nx \Big|_0^{\pi} - \int_0^{\pi} \cos nxdx) = \frac{4}{n^2\pi} [(-1)^n \pi] = (-1)^n \frac{4}{n^2}, \end{aligned}$$

$$b_n = 0,$$

$$f(x) = \frac{\pi^2}{3} + 4 \sum_{n=1}^{\infty} (-1)^n \frac{\cos nx}{n^2}, \quad -\pi < x < \pi$$

