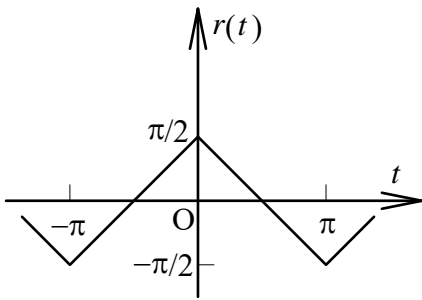


Find the Fourier series of the periodic function $r(t)$ of period $p=2\pi$, as shown below. [96 中央機械二 2]



$$[\text{解}] r(t) = \begin{cases} -\frac{\pi}{2} + t, & -\pi < t < 0 \\ \frac{\pi}{2} - t, & 0 < t < \pi \end{cases} \Rightarrow r(t) \text{ 為偶函數} \Rightarrow r(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nt$$

$$a_0 = \frac{2}{\pi} \int_0^{\pi} r(t) dt = \frac{2}{\pi} \int_0^{\pi} \left(\frac{\pi}{2} - t\right) dt = \frac{2}{\pi} \cdot \left(\frac{\pi}{2}t - \frac{t^2}{2}\right) \Big|_0^{\pi} = \frac{2}{\pi} \cdot \left(\frac{\pi^2}{2} - \frac{\pi^2}{2}\right) = 0$$

$$a_n = \frac{2}{\pi} \int_0^{\pi} r(t) \cos ntdt = \frac{2}{\pi} \int_0^{\pi} \left(\frac{\pi}{2} - t\right) \cos ntdt = \frac{2}{n\pi} \left[\left(\frac{\pi}{2} - t\right) \sin nt \Big|_0^{\pi} + \int_0^{\pi} \sin ntdt \right]$$

$$= \frac{2}{n\pi} \left(0 - \frac{\cos nt}{n} \Big|_0^{\pi} \right) = -\frac{2}{n^2\pi} (\cos n\pi - 1) = -\frac{2}{n^2\pi} [(-1)^n - 1] = \frac{4}{(2n-1)^2\pi}$$

$$r(t) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\cos(2n-1)t}{(2n-1)^2}$$