

Please find the Fourier series of the periodic function: $f(x) = x, |x| < T$. [95中正機械2(b)]

[解] $f(x)$ 為奇函數，週期為 $2T \Rightarrow$ 設 $f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{T}$

$$b_n = \frac{2}{T} \int_0^T f(x) \sin \frac{n\pi x}{T} dx = \frac{2}{T} \int_0^T x \sin \frac{n\pi x}{T} dx = -\frac{2}{n\pi} \left(x \cos \frac{n\pi x}{T} \Big|_0^T - \int_0^T \cos \frac{n\pi x}{T} dx \right)$$

$$= -\frac{2}{n\pi} \left(T \cos n\pi - \frac{T}{n\pi} \sin \frac{n\pi x}{T} \Big|_0^T \right) = -(-1)^n \frac{2T}{n\pi} = (-1)^{n+1} \frac{2T}{n\pi}$$

$$f(x) = \frac{2T}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin \frac{n\pi x}{T}$$

南臺科技大學

Southern Taiwan University of Science and Technology