Expand $f(x)=\begin{cases}-1, & -\pi < x < 0 \\ 1, & 0 \le x < \pi\end{cases}$ in a complex Fourier series. [93 中央機械 8(b)]

[解]令 $f(x)=\sum_{n=-\infty}^{\infty} c_n e^{inx}, \quad -\pi < x < \pi$

$$c_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)e^{-inx}dx = \frac{1}{2\pi}[\int_{-\pi}^{0} -1\cdot e^{-inx}dx + \int_{0}^{\pi} 1\cdot e^{-inx}dx] = \frac{1}{2\pi}(\frac{e^{-inx}}{in}\Big|_{-\pi}^{0} + \frac{e^{-inx}}{-in}\Big|_{0}^{\pi})$$

$$= \frac{1}{2\pi}(\frac{1-e^{in\pi}}{in} + \frac{e^{-in\pi}-1}{-in}) = \frac{1}{2\pi}(\frac{2}{in} - \frac{e^{in\pi}}{in} - \frac{e^{-in\pi}}{in})$$

$$= \frac{1}{2\pi}[\frac{2}{in} - \frac{\cos n\pi + i\sin n\pi}{in} - \frac{\cos(-n\pi) + i\sin(-n\pi)}{in}]$$

$$= \frac{1}{2\pi}[\frac{2}{in} - \frac{(-1)^n}{in} - \frac{(-1)^n}{in}] = \frac{1}{\pi}[\frac{1-(-1)^n}{in}] = \frac{2}{i\pi(2n-1)}$$

$$\therefore f(x) = \frac{2}{i\pi}\sum_{n=-\infty}^{\infty} \frac{1}{2n-1} e^{i(2n-1)x}, \quad -\pi < x < \pi$$