Let $f(x)=e^{-|x|}$, compute the complex Fourier integral of $f(x)$. Note that $f(x)=\begin{cases} e^{-x}, & x\geq 0 \\ e^{x}, & x<0 \end{cases}$. [88 成大機械 5]

[解]令$f(x)=\int_{-\infty}^{\infty} c(\omega)e^{i\omega x}d\omega$

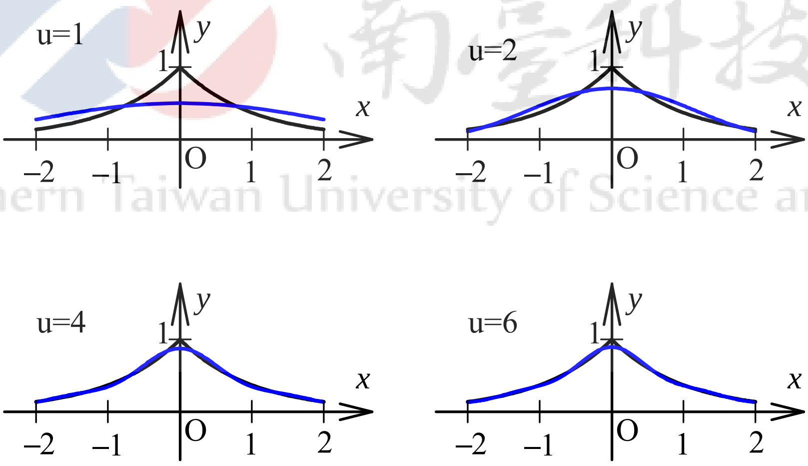
$$c(\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty} f(x)e^{-i\omega x}dx=\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-|x|}e^{-i\omega x}dx=\frac{1}{2\pi}(\int_{-\infty}^{0} e^{x}e^{-i\omega x}dx+\int_{0}^{\infty} e^{-x}e^{-i\omega x}dx)$$

$$=\frac{1}{2\pi}(\int_{-\infty}^{0} e^{(1-i\omega)x}dx+\int_{0}^{\infty} e^{-(1+i\omega)x}dx)=\frac{1}{2\pi}(\frac{e^{(1-i\omega)x}}{1-i\omega}\Big|_{-\infty}^{0}-\frac{e^{-(1+i\omega)x}}{1+i\omega}\Big|_{0}^{\infty})$$

$$=\frac{1}{2\pi}(\frac{1-0}{1-i\omega}-\frac{0-1}{1+i\omega})=\frac{1}{2\pi}\cdot\frac{(1+i\omega)+(1-i\omega)}{(1-i\omega)(1+i\omega)}=\frac{1}{\pi(1+\omega^2)}$$

$$f(x)=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{1}{1+\omega^2}e^{i\omega x}d\omega$$

以 $f(x)=\frac{1}{\pi}\int_{-u}^{u}\frac{1}{1+\omega^2}e^{i\omega x}d\omega$畫圖，如下




南臺科技大學
Southern Taiwan University of Science and Technology