Find the Fourier cosine series and Fourier sine series of $f(x)$, where $f(x)=\sin x$, $0<x<\pi$. [106暨南應光5]

[解](1)設 $f(x)=\dfrac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos nx$

$$a_0=\frac{2}{\pi}\int_0^{\pi}f(x)dx=\frac{2}{\pi}\int_0^{\pi}\sin xdx=\frac{2}{\pi}\cdot(-\cos x)\Big|_0^{\pi}=\frac{2}{\pi}\cdot(\cos\pi-1)=-\frac{4}{\pi}$$

$$a_n=\frac{2}{\pi}\int_0^{\pi}f(x)\cos nxdx=\frac{2}{\pi}\int_0^{\pi}\sin x\cos nxdx=\frac{1}{\pi}\int_0^{\pi}[\sin(1+n)x+\sin(1-n)x]dx$$

$$=-\frac{1}{\pi}[\frac{\cos(1+n)x}{1+n}+\frac{\cos(1-n)x}{1-n}]\Big|_0^{\pi}=-\frac{1}{\pi}[\frac{\cos(1+n)\pi-1}{1+n}+\frac{\cos(1-n)\pi-1}{1-n}]$$

$$=-\frac{1}{\pi}[\frac{(-1)^{1+n}-1}{1+n}+\frac{(-1)^{1-n}-1}{1-n}]=-\frac{1}{\pi}(\frac{-2}{1+2n}+\frac{-2}{1-2n})=\frac{2}{\pi}(\frac{1}{1+2n}+\frac{1}{1-2n})=\frac{4}{(1-4n^2)\pi}$$

$$f(x)=-\frac{2}{\pi}+\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{1-4n^2}\cos 2nx,\ 0<x<\pi$$

(2)設 $f(x)=\sum_{n=1}^{\infty}b_n\sin nx$, 其中$b_1=1, b_n=0, n\neq 1$

$$f(x)=\sin x,\ 0<x<\pi$$