There is a periodic square wave with analytically represented as $f(x)$ function $f(x)=\begin{cases}-k, & \text{if } -\pi<x<0\\ k, & \text{if } 0<x<\pi\end{cases}$, and $f(x+2\pi)=f(x)$. Please find the Fourier coefficients a_n, b_n and their series function to present the $f(x)$ function. [106 元智機械 7]

Solution : $f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n\cos nx+b_n\sin nx)$,

$$a_0=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)dx=0,$$

$$a_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\cos nxdx=0,$$

$$b_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\sin nxdx=\frac{2}{\pi}\int_{0}^{\pi}k\sin nxdx=\frac{2k}{n\pi}[1-(-1)^n]$$

$$=\begin{cases}0, & \text{for even } n\\ \frac{4k}{n\pi}, & \text{for odd } n\end{cases}=\frac{4k}{(2n-1)\pi}$$

$$f(x)=\frac{4k}{\pi}\sum_{n=1}^{\infty}\frac{1}{2n-1}\sin(2n-1)x.$$

The graph of the Fourier series is shown as follows: