

Find the Fourier series of the given function $f(x)$, which is assumed to have the period 2π .

$$f(x) = \begin{cases} x + \pi, & \text{if } -\pi < x < 0 \\ -x + \pi, & \text{if } 0 < x < \pi \end{cases}. \quad [106 \text{中山材光6}]$$

[解] 令 $F(x) = f(x) - \pi$, 則 $F(x)$ 為奇函數, 其 Fourier 級數為

$$F(x) = \sum_{n=1}^{\infty} b_n \sin nx, \text{ with}$$

$$\begin{aligned} b_n &= \frac{2}{\pi} \int_0^{\pi} x \sin nxdx = \frac{2}{\pi} \left(-\frac{1}{n} \right) \left(x \cos nx \Big|_0^{\pi} - \int_0^{\pi} \cos nxdx \right) \\ &= -\frac{2}{n\pi} (\pi \cos n\pi) = -\frac{2}{n} (-1)^n \end{aligned}$$

$$F(x) = -2 \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin nx \Rightarrow f(x) = \pi - 2 \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin nx$$

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