

Find the Fourier series of $f(x)$ as given over one period. $f(x) = x^2$ ($-\frac{\pi}{2} < x < \frac{\pi}{2}$). [105成大造船2]

[解] $f(x)$ 為偶函數，週期為 $\pi \Rightarrow$ 設 $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos 2nx$

$$a_0 = \frac{4}{\pi} \int_0^{\frac{\pi}{2}} f(x) dx = \frac{4}{\pi} \int_0^{\frac{\pi}{2}} x^2 dx = \frac{4}{\pi} \cdot \frac{x^3}{3} \Big|_0^{\frac{\pi}{2}} = \frac{4}{\pi} \cdot \frac{\pi^3}{24} = \frac{\pi^2}{6}$$

$$a_n = \frac{4}{\pi} \int_0^{\frac{\pi}{2}} f(x) \cos 2nxdx = \frac{4}{\pi} \int_0^{\frac{\pi}{2}} x^2 \cos 2nxdx = \frac{4}{\pi} \cdot \frac{1}{2n} (x^2 \sin 2nx \Big|_0^{\frac{\pi}{2}} - 2 \int_0^{\frac{\pi}{2}} x \sin 2nxdx)$$
$$= \frac{2}{n\pi} [2 \cdot \frac{1}{2n} (x \cos 2nx \Big|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \cos 2nxdx)] = \frac{2}{n\pi} (\frac{1}{n} \cdot \frac{\pi}{2} \cos n\pi) = \frac{(-1)^n}{n^2}$$

$$f(x) = \frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos 2nx$$