Find the Fourier transform of the function $f(x)=\begin{cases} x, & \text{if } 0<x<a \\ 0, & \text{otherwise} \end{cases}$. [105 彰師光電 3]

[解] $\int_{-\infty}^{\infty} f(x)e^{-i\omega x}dx = \int_0^a xe^{-i\omega x}dx = -\frac{1}{i\omega}[xe^{-i\omega x}\Big|_0^a - \int_0^a e^{-i\omega x}dx] = -\frac{1}{i\omega}[ae^{-i\omega a} + \frac{e^{-i\omega x}}{i\omega}\Big|_0^a]$

$$= -\frac{1}{i\omega}[ae^{-i\omega a} + \frac{e^{-i\omega a}-1}{i\omega}] = \frac{i\omega a e^{-i\omega a} + e^{-i\omega a} - 1}{\omega^2}$$

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