

Find the Fourier series of the function $f(x) = x + 5, -1 < x < 1, f(x) = f(x + 2)$. [105 南大綠能 7]

[解] 令 $F(x) = f(x) - 5 = x$, 則 $F(x)$ 為奇函數 $\Rightarrow F(x) = \sum_{n=1}^{\infty} b_n \sin n\pi x$

$$b_n = 2 \int_0^1 f(x) \sin n\pi x dx = 2 \int_0^1 x \sin n\pi x dx$$

$$= -\frac{2}{n\pi} (x \cos n\pi x \Big|_0^1 - \int_0^1 \cos n\pi x dx)$$

$$= -\frac{2}{n\pi} [(\cos n\pi - 0) - 0] = -\frac{2(-1)^n}{n\pi}$$

$$F(x) = -\frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin n\pi x \Rightarrow f(x) = 5 - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin n\pi x$$

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