

Find the Fourier series of the function, periodic square wave $f(t) = \begin{cases} 0, & -2 < t < -1 \\ k, & -1 < t < 1 \\ 0, & 1 < t < 2 \end{cases}$. [105元智機

械6]

[解] $f(t)$ 為偶函數 \Rightarrow 設 $f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi t}{2}$

$$a_0 = \frac{1}{2} \int_{-2}^2 f(t) dt = \int_0^1 k dt = kt \Big|_0^1 = k$$

$$a_n = \frac{1}{2} \int_{-2}^2 f(t) \cos \frac{n\pi t}{2} dt = \int_0^1 k \cos \frac{n\pi t}{2} dt = \frac{2k}{n\pi} \cdot \sin \frac{n\pi t}{2} \Big|_0^1 = \frac{2k}{n\pi} \sin \frac{n\pi}{2} = \frac{2k}{(2n-1)\pi} (-1)^{n-1}$$

$$f(t) = \frac{k}{2} + \frac{2k}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos \frac{n\pi t}{2}$$