

Transform the function $f(x) = \begin{cases} x, & \text{if } 0 < x < a \\ 0, & \text{if } x > a \end{cases}$ in the form of sine integral. [105 中正光電 6]

$$\begin{aligned} \text{[解]} F_S(\omega) &= \int_0^{\infty} f(x) \sin \omega x dx = \int_0^a x \sin \omega x dx = -\frac{1}{\omega} (x \cos \omega x) \Big|_0^a - \int_0^a \cos \omega x dx \\ &= -\frac{1}{\omega} \left(a \cos \omega a - \frac{\sin \omega x}{\omega} \Big|_0^a \right) = -\frac{1}{\omega} \left(a \cos \omega a - \frac{\sin \omega a}{\omega} \right) = \frac{\sin \omega a - \omega a \cos \omega a}{\omega^2} \end{aligned}$$

$$f(x) = \frac{2}{\pi} \int_0^{\infty} \frac{\sin \omega a - \omega a \cos \omega a}{\omega^2} \sin \omega x d\omega$$

南臺科技大學

Southern Taiwan University of Science and Technology