試求函數 $f(x)$ 的傅立葉餘弦級數(Fourier cosine series)。$f(x)=x,\ 0<x<\pi$ [104 高第一環安衛 7]

[解]設 $f(x)=\dfrac{a_0}{2}+\displaystyle\sum_{n=1}^{\infty}a_n\cos nx$, 其中

$$a_0=\frac{2}{\pi}\int_0^\pi f(x)dx=\frac{2}{\pi}\int_0^\pi xdx=\frac{2}{\pi}\cdot\left.\frac{x^2}{2}\right|_0^\pi=\pi$$

$$a_n=\frac{2}{\pi}\int_0^\pi f(x)\cos nxdx=\frac{2}{\pi}\int_0^\pi x\cos nxdx=\frac{2}{n\pi}\left(\left.x\sin nx\right|_0^\pi-\int_0^\pi \sin nxdx\right)$$

$$=\left.\frac{2\cos nx}{n^2\pi}\right|_0^\pi=\frac{2(\cos n\pi-1)}{n^2\pi}=\frac{2[(-1)^n-1]}{n^2\pi}=\frac{-4}{(2n-1)^2\pi}$$

$$f(x)=\frac{\pi}{2}-\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{\cos(2n-1)x}{(2n-1)^2},\quad 0<x<\pi$$