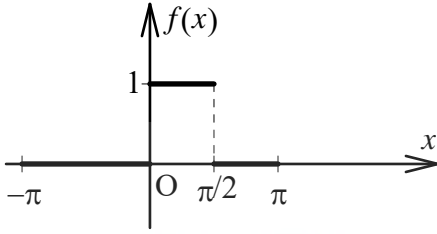Find the Fourier series of the following function $f(x)$, which is assumed to have the period 2π.

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[解] $f(x) = \dfrac{a_0}{2} + \sum\limits_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$,

$a_0 = \dfrac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx = \dfrac{1}{\pi}\int_{0}^{\pi/2} 1 \cdot dx = \dfrac{1}{\pi} \cdot x\Big|_0^{\pi/2} = \dfrac{1}{\pi} \cdot \dfrac{\pi}{2} = \dfrac{1}{2}$

$a_n = \dfrac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx = \dfrac{1}{\pi}\int_{0}^{\pi/2} 1 \cdot \cos nx\,dx = \dfrac{1}{n\pi} \cdot \sin nx\Big|_0^{\pi/2} = \dfrac{1}{n\pi} \cdot \sin\dfrac{n\pi}{2}$

$b_n = \dfrac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx = \dfrac{1}{\pi}\int_{0}^{\pi/2} 1 \cdot \sin nx\,dx = -\dfrac{1}{n\pi} \cdot \cos nx\Big|_0^{\pi/2} = -\dfrac{1}{n\pi}(\cos\dfrac{n\pi}{2} - 1)$

$f(x) = \dfrac{1}{4} + \dfrac{1}{\pi}\sum\limits_{n=1}^{\infty}\dfrac{1}{n}[\sin\dfrac{n\pi}{2}\cos nx - (\cos\dfrac{n\pi}{2} - 1)\sin nx]$