

已知 $f(x)=1$ ， $0 < x < \pi$ ，請完成下列工作，(a)寫出餘弦半幅擴張 $\frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos nx$ ，其中 $a_0 = \frac{2}{\pi} \int_0^{\pi} f(x) dx$ ， $a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nxdx$ ， $n=1, 2, 3, \dots$ ；(b)寫出正弦半幅擴張 $\sum_{n=1}^{\infty} b_n \sin nx$ ，其中 $b_n = \frac{2}{\pi} \int_0^{\pi} f(x) \sin nxdx$ ， $n=1, 2, 3, \dots$ 。 [104高海輪機七]

$$[\text{解}](a) f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$$

$$a_0 = \frac{2}{\pi} \int_0^{\pi} f(x) dx = \frac{2}{\pi} \int_0^{\pi} 1 \cdot dx = \frac{2}{\pi} \cdot x \Big|_0^{\pi} = 2$$

$$a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nxdx = \frac{2}{\pi} \int_0^{\pi} 1 \cdot \cos nxdx = \frac{2}{n\pi} \sin nx \Big|_0^{\pi} = 0$$

$$f(x) = 1$$

$$(b) f(x) = \sum_{n=1}^{\infty} b_n \sin nx$$

$$b_n = \frac{2}{\pi} \int_0^{\pi} f(x) \sin nxdx = \frac{2}{\pi} \int_0^{\pi} 1 \cdot \sin nxdx = -\frac{2}{n\pi} \cos nx \Big|_0^{\pi} = -\frac{2}{n\pi} (\cos n\pi - 1)$$

$$= -\frac{2}{n\pi} [(-1)^n - 1] = \frac{4}{(2n-1)\pi}$$

$$f(x) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\sin(2n-1)x}{2n-1}$$