

Find the Fourier integral representation of the following non-periodic function:

$$f(\theta) = \begin{cases} \cos \theta, & -\frac{\pi}{2} < \theta < \frac{\pi}{2} \\ 0, & \text{otherwise} \end{cases} \cdot [104 \text{ 高師大電子 5}]$$

[解] 令 $f(\theta) = \int_0^{\infty} [a(\omega) \cos \omega \theta + b(\omega) \sin \omega \theta] d\omega$, $f(\theta)$ 為偶函數 $\Rightarrow b(\omega) = 0$

$$\begin{aligned} a(\omega) &= \frac{2}{\pi} \int_0^{\infty} f(\theta) \cos \omega \theta d\theta = \frac{2}{\pi} \int_0^{\frac{\pi}{2}} \cos \theta \cos \omega \theta d\theta \\ &= \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \cos(1+\omega)\theta + \cos(1-\omega)\theta d\theta = \frac{1}{\pi} \cdot \left[\frac{\sin(1+\omega)\theta}{1+\omega} + \frac{\sin(1-\omega)\theta}{1-\omega} \right] \Big|_0^{\frac{\pi}{2}} \\ &= \frac{1}{\pi} \left[\frac{\sin \frac{(1+\omega)\pi}{2}}{1+\omega} + \frac{\sin \frac{(1-\omega)\pi}{2}}{1-\omega} \right] = \frac{1}{\pi} \frac{(1-\omega) \sin \frac{(1+\omega)\pi}{2} + (1+\omega) \sin \frac{(1-\omega)\pi}{2}}{1-\omega^2} \\ &= \frac{1}{\pi} \frac{[\sin \frac{(1+\omega)\pi}{2} + \sin \frac{(1-\omega)\pi}{2}] - \omega [\sin \frac{(1+\omega)\pi}{2} - \sin \frac{(1-\omega)\pi}{2}]}{1-\omega^2} \\ &= \frac{1}{\pi} \frac{2 \sin \frac{\pi}{2} \cos \frac{\omega\pi}{2} - 2\omega \cos \frac{\pi}{2} \sin \frac{\omega\pi}{2}}{1-\omega^2} = \frac{1}{\pi} \frac{2 \cdot 1 \cdot \cos \frac{\omega\pi}{2} - 2\omega \cdot 0 \cdot \sin \frac{\omega\pi}{2}}{1-\omega^2} = \frac{2 \cos \frac{\omega\pi}{2}}{\pi(1-\omega^2)} \\ f(\theta) &= \frac{2}{\pi} \int_0^{\infty} \frac{\cos \frac{\omega\pi}{2}}{1-\omega^2} \cos \omega \theta d\omega \end{aligned}$$

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