Let a and k be positive numbers, and let $f(t) = \begin{cases} k, & -a \le t < a \\ 0, & t < -a, t \ge a \end{cases}$. Find the Fourier transform of $f(t)$.

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[解] $\displaystyle\int_{-\infty}^{\infty} f(t)e^{-i\omega t}\,dt = \int_{-a}^{a} ke^{-i\omega t}\,dt = \frac{k}{-i\omega} \cdot e^{-i\omega t}\Big|_{-a}^{a} = \frac{k}{-i\omega}(e^{-i\omega a} - e^{i\omega a})$

$\displaystyle = \frac{k}{-i\omega}(-2i\sin a\omega) = \frac{2k\sin a\omega}{\omega}$