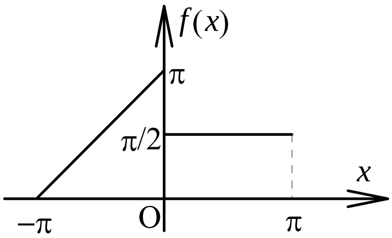
Find the Fourier series of the given function as shown, which is assumed to have the periodic 2π. [104 中原機械丙 6]



[解] $f(x)=\begin{cases}\pi+x, & -\pi<x<0\\ \dfrac{\pi}{2}, & 0<x<\pi\end{cases}$

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n\cos nx+b_n\sin nx),$$

$$a_0=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)dx=\frac{1}{\pi}[\int_{-\pi}^{0}(\pi+x)dx+\int_0^{\pi}\frac{\pi}{2}dx]=\frac{1}{\pi}[(\pi x+\frac{x^2}{2})\Big|_{-\pi}^{0}+\frac{\pi}{2}x\Big|_0^{\pi}]=\pi$$

$$a_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\cos nxdx=\frac{1}{\pi}[\int_{-\pi}^{0}(\pi+x)\cos nxdx+\int_0^{\pi}\frac{\pi}{2}\cos nxdx]$$

$$=\frac{1}{\pi}[\frac{1}{n}(\pi\sin nx\Big|_{-\pi}^{0}+x\sin nx\Big|_{-\pi}^{0}-\int_{-\pi}^{0}\sin nxdx+\frac{\pi}{2}\sin nx\Big|_0^{\pi})]$$

$$=\frac{1}{\pi}[\frac{1}{n}(0+0+\frac{\cos nx}{n}\Big|_{-\pi}^{0}+0)]=\frac{1-\cos n\pi}{n^2\pi}=\frac{1-(-1)^n}{n^2\pi}$$

$$b_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\sin nxdx=\frac{1}{\pi}[\int_{-\pi}^{0}(\pi+x)\sin nxdx+\int_0^{\pi}\frac{\pi}{2}\sin nxdx]$$

$$=\frac{1}{\pi}[-\frac{1}{n}(\pi\cos nx\Big|_{-\pi}^{0}+x\cos nx\Big|_{-\pi}^{0}-\int_{-\pi}^{0}\cos nxdx+\frac{\pi}{2}\cos nx\Big|_0^{\pi})]$$

$$=\frac{1}{\pi}\{-\frac{1}{n}[\pi(1-\cos n\pi)+(0+\pi\cos n\pi)-0+\frac{\pi}{2}(\cos n\pi-1)]\}=-\frac{1}{n\pi}[\pi+\frac{\pi}{2}(\cos n\pi-1)]$$

$$=-\frac{1}{n\pi}\{\pi+\frac{\pi}{2}[(-1)^n-1]\}$$

$$f(x)=\frac{\pi}{2}+\frac{1}{\pi}\sum_{n=1}^{\infty}\left\{\frac{1-(-1)^n}{n^2}\cos nx-\frac{1}{n}\{\pi+\frac{\pi}{2}[(-1)^n-1]\}\sin nx\right\}$$