Find the Fourier cosine transformation of $f(x)=e^{-x}$. [103 高海電訊 4]

[解]先推導$\int e^{ax}e^{ibx}dx=\int e^{(a+ib)x}dx=\frac{e^{(a+ib)x}}{a+ib}=\frac{(a-ib)e^{(a+ib)x}}{a^2+b^2}=\frac{e^{ax}(a-ib)(\cos bx+i\sin bx)}{a^2+b^2}$

$$=\frac{e^{ax}[(a\cos bx+b\sin bx)+i(a\sin bx-b\cos bx)]}{a^2+b^2}$$

實部$\int e^{ax}\cos bxdx=\frac{e^{ax}(a\cos bx+b\sin bx)}{a^2+b^2}$, 虛部$\int e^{ax}\sin bxdx=\frac{e^{ax}(a\sin bx-b\cos bx)}{a^2+b^2}$

$f(x)$的 Fourier cosine 轉換為

$$F_c(\omega)=\int_0^\infty f(x)\cos\omega xdx=\int_0^\infty e^{-x}\cos\omega xdx=\left.\frac{e^{-x}(-\cos\omega x+\omega\sin\omega x)}{(-1)^2+\omega^2}\right|_0^\infty$$

$$=0-\frac{1\cdot(-1+0)}{1+\omega^2}=\frac{1}{1+\omega^2}$$