

Expand the given function in an appropriate cosine or sine Fourier series. $f(x) = |\sin x|$, $-\pi < x < \pi$.

[103逢甲電機4]

[解] $f(x)$ 為偶函數，其 Fourier 級數為

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$$

$$a_0 = \frac{2}{\pi} \int_0^{\pi} (-x + \pi) dx = \frac{2}{\pi} \cdot \left(-\frac{x^2}{2} + \pi x \right) \Big|_0^{\pi} = \frac{2}{\pi} \cdot \left(-\frac{\pi^2}{2} + \pi^2 \right) = \pi$$

$$a_n = \frac{2}{\pi} \int_0^{\pi} (-x + \pi) \cos nx dx = \frac{2}{\pi} \left(\frac{1}{n} \right) \left((-x + \pi) \sin nx \Big|_0^{\pi} + \int_0^{\pi} \sin nx dx \right)$$

$$= \frac{2}{n\pi} \left(0 - \frac{\cos nx}{n} \Big|_0^{\pi} \right) = -\frac{2}{n^2\pi} (\cos n\pi - 1) = -\frac{2}{n^2\pi} [(-1)^n - 1] = \frac{4}{(2n-1)^2\pi}$$

$$f(x) = \frac{\pi}{2} + \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} \cos(2n-1)x$$