

Find the Fourier series of the function on the interval $[-\pi, \pi]$, $f(x) = -1, -\pi \leq x \leq 0$, and $f(x) = +1, 0 \leq x \leq \pi$. [103 海洋機械機電 6]

$$[\text{解}] f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx),$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx = 0$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nxdx = 0$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nxdx = \frac{2}{\pi} \int_0^{\pi} 1 \cdot \sin nxdx = \frac{2}{n\pi} [1 - (-1)^n]$$

$$= \begin{cases} 0, & n \text{ 為偶數} \\ \frac{4}{n\pi}, & n \text{ 為奇數} \end{cases} = \frac{4}{(2n-1)\pi}$$

$$f(x) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin(2n-1)x, \quad -\pi < x < \pi$$