

There is periodic square wave with analytic with represented as $f(x)$ function

$$f(x) = \begin{cases} -k, & \text{when } -\pi < x < 0 \\ k, & \text{when } 0 < x < \pi \end{cases}, \text{ and } f(x+2\pi) = f(x). \text{ Please find the Fourier coefficients } a_n, b_n \text{ and}$$

their series functions to represent the $f(x)$ functions. [103元智機械7]

[解] $f(x)$ 為奇函數 \Rightarrow 設 $f(x) = \sum_{n=1}^{\infty} b_n \sin nx$

$$b_n = \frac{2}{\pi} \int_0^{\pi} k \sin nx dx = -\frac{2k}{n\pi} \cdot \cos nx \Big|_0^{\pi} = -\frac{2k}{n\pi} \cdot (\cos n\pi - 1) = -\frac{2k}{n\pi} \cdot [(-1)^n - 1] = \frac{4k}{(2n-1)\pi}$$

$$f(x) = \frac{4k}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin(2n-1)x$$

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