

Find the Fourier series of the given function  $f(x)$ , which is assumed to have the period  $2\pi$ .

$$f(x) = \begin{cases} x + \pi, & \text{if } -\pi < x < 0 \\ -x + \pi, & \text{if } 0 < x < \pi \end{cases}. \quad [103 \text{ 中山材光6}]$$

[解]  $f(x)$  為偶函數，其 Fourier 級數為

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$$

$$a_0 = \frac{2}{\pi} \int_0^{\pi} (-x + \pi) dx = \frac{2}{\pi} \cdot \left(-\frac{x^2}{2} + \pi x\right) \Big|_0^{\pi} = \frac{2}{\pi} \cdot \left(-\frac{\pi^2}{2} + \pi^2\right) = \pi$$

$$\begin{aligned} a_n &= \frac{2}{\pi} \int_0^{\pi} (-x + \pi) \cos nx dx = \frac{2}{\pi} \left(\frac{1}{n}\right) \left( (-x + \pi) \sin nx \Big|_0^{\pi} + \int_0^{\pi} \sin nx dx \right) \\ &= \frac{2}{n\pi} \left(0 - \frac{\cos nx}{n} \Big|_0^{\pi}\right) = -\frac{2}{n^2\pi} (\cos n\pi - 1) = -\frac{2}{n^2\pi} [(-1)^n - 1] = \frac{4}{(2n-1)^2\pi} \end{aligned}$$

$$f(x) = \frac{\pi}{2} + \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} \cos(2n-1)x$$

