

$f(x) = x^2, 0 < x < 2\pi, f(x) = f(x + 2\pi)$ . Find the Fourier series. [103 中央機械 6(a)]

$$[\text{解}] f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx),$$

$$a_0 = \frac{1}{\pi} \int_0^{2\pi} x^2 dx = \frac{1}{\pi} \cdot \frac{x^3}{3} \Big|_0^{2\pi} = \frac{8\pi^2}{3}$$

$$a_n = \frac{1}{\pi} \int_0^{2\pi} x^2 \cos nxdx = \frac{1}{n\pi} (x^2 \sin nx \Big|_0^{2\pi} - \int_0^{2\pi} 2x \sin nxdx)$$
$$= \frac{2}{n^2\pi} (x \cos nx \Big|_0^{2\pi} - \int_0^{2\pi} \cos nxdx) = \frac{2}{n^2\pi} (2\pi) = \frac{4}{n^2}$$

$$b_n = \frac{1}{\pi} \int_0^{2\pi} x^2 \sin nxdx = -\frac{1}{n\pi} (x^2 \cos nx \Big|_0^{2\pi} - \int_0^{2\pi} 2x \cos nxdx)$$

$$= -\frac{1}{n\pi} [4\pi^2 - \frac{2}{n} (x \sin nx \Big|_0^{2\pi} - \int_0^{2\pi} \sin nxdx)] = -\frac{1}{n\pi} (4\pi^2 + \frac{2}{n^2} \cdot \cos nx \Big|_0^{2\pi}) = -\frac{4\pi}{n}$$

$$f(x) = \frac{4\pi^2}{3} + 4 \sum_{n=1}^{\infty} \left( \frac{\cos nx}{n^2} - \frac{\pi \sin nx}{n} \right)$$