Find the Fourier series of the function $f(x) = \begin{cases} -k, & -1 < x < 0 \\ k, & 0 < x < 1 \end{cases}$. [103中原機械丙8]

[解]$f(x)$為奇函數 $\Rightarrow$ 令$f(x) = \displaystyle\sum_{n=1}^{\infty} b_n \sin n\pi x$

$$b_n = 2\int_0^1 f(x)\sin n\pi x\, dx = 2\int_0^1 k\sin n\pi x\, dx = -2k\left.\frac{\cos n\pi x}{n\pi}\right|_0^1 = -2k\frac{\cos n\pi - 1}{n\pi}$$

$$= -2k\frac{(-1)^n - 1}{n\pi} = 2k\frac{1 - (-1)^n}{n\pi} = \frac{4k}{(2n-1)\pi}$$

$$f(x) = \frac{4k}{\pi}\sum_{n=1}^{\infty}\frac{1}{2n-1}\sin(2n-1)\pi x$$

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