

對一函數 $f(x)$ ， $-L \leq x \leq L$ ，可用下列的傅立葉級數展開：

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right) ; \text{若 } f(x) = \begin{cases} -2, & -\pi \leq x \leq 0 \\ 2, & 0 < x \leq \pi \end{cases}, (L = \pi), \text{ 求 } a_0, a_n \text{ 及 } b_n。$$

[102 虎尾電子 5]

[解] $f(x)$ 為奇函數 $\Rightarrow a_0 = a_n = 0$ ，且 $f(x) = \sum_{n=1}^{\infty} b_n \sin nx$

$$\begin{aligned} b_n &= \frac{2}{\pi} \int_0^{\pi} f(x) \sin nx dx = \frac{2}{\pi} \int_0^{\pi} 2 \sin nx dx = -\frac{4}{n\pi} \cdot \cos nx \Big|_0^{\pi} \\ &= -\frac{4}{n\pi} (\cos n\pi - 1) = -\frac{4}{n\pi} [(-1)^n - 1] = \frac{8}{(2n-1)\pi} \end{aligned}$$

Southern Taiwan University of Science and Technology