

Find the Fourier series of periodic function $f(x)$, $f(x) = -1(-1 < x < 0)$, $f(x) = 1(0 < x < 1)$, $P = 2L = 2$.

[102中原機械乙5]

[解] $f(x)$ 為奇函數 \Rightarrow 令 $f(x) = \sum_{n=1}^{\infty} b_n \sin n\pi x$

$$b_n = 2 \int_0^1 f(x) \sin n\pi x dx = 2 \int_0^1 1 \cdot \sin n\pi x dx = -2 \left. \frac{\cos n\pi x}{n\pi} \right|_0^1 = -2 \frac{\cos n\pi - 1}{n\pi}$$

$$= -2 \frac{(-1)^n - 1}{n\pi} = 2 \frac{1 - (-1)^n}{n\pi} = \frac{4}{(2n-1)\pi}$$

$$f(x) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin(2n-1)\pi x$$

Southern Taiwan University of Science and Technology