

$f(x) = \begin{cases} k, & -1 \leq x < 1 \\ 0, & \text{otherwise} \end{cases}$, (a)求 $f(x)$ 的傅立葉轉換(Fourier transform) (b)求 $\int_{-\infty}^{\infty} \frac{\sin^2 \omega}{\omega^2} d\omega$. [101 虎尾電機 3]

$$[\text{解}](a) F(\omega) = \int_{-\infty}^{\infty} f(x)e^{-i\omega x} dx = \int_{-1}^1 ke^{-i\omega x} dx = k \cdot \frac{e^{-i\omega} - e^{i\omega}}{-i\omega} = \frac{2k \sin \omega}{\omega}$$

(b)由 Parseval's 關係式得

$$\int_{-\infty}^{\infty} |f(x)|^2 dx = \frac{1}{2\pi} \int_{-\infty}^{\infty} |F(\omega)|^2 d\omega \Rightarrow \int_{-1}^1 k^2 \cdot dx = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left(\frac{2k \sin \omega}{\omega} \right)^2 d\omega$$
$$2k^2 = \frac{2k^2}{\pi} \int_{-\infty}^{\infty} \frac{\sin^2 \omega}{\omega^2} d\omega \Rightarrow \int_{-\infty}^{\infty} \frac{\sin^2 \omega}{\omega^2} d\omega = \pi$$

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