一函數具有週期性：$f(x)=f(x+2\pi)$，其在$-\pi \le x \le \pi$之區間內定義為 $f(x)=\begin{cases} 2, & -\pi \le x < 0 \\ 1, & 0 \le x < \pi \end{cases}$. [101 中原土木 5]

[解] $f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$,

$$a_0=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx=\frac{1}{\pi}(\int_{-\pi}^{0} 2dx+\int_{0}^{\pi} 1dx)=\frac{1}{\pi}(2x\Big|_{-\pi}^{0}+x\Big|_{0}^{\pi})=3$$

$$a_n=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nxdx=\frac{1}{\pi}(\int_{-\pi}^{0} 2\cos nxdx+\int_{0}^{\pi}\cos nxdx)=\frac{1}{\pi}(\frac{2\sin nx}{n}\Big|_{-\pi}^{0}+\frac{\sin nx}{n}\Big|_{0}^{\pi})=0$$

$$b_n=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nxdx=\frac{1}{\pi}(\int_{-\pi}^{0} 2\sin nxdx+\int_{0}^{\pi}\sin nxdx)=-\frac{1}{\pi}(\frac{2\cos nx}{n}\Big|_{-\pi}^{0}+\frac{\cos nx}{n}\Big|_{0}^{\pi})$$

$$=-\frac{1}{\pi}[\frac{2(1-\cos n\pi)}{n}+\frac{\cos n\pi-1}{n}]=\frac{\cos n\pi-1}{n\pi}=\frac{(-1)^n-1}{n\pi}=-\frac{2}{(2n-1)\pi}$$

$$f(x)=\frac{3}{2}-\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{2n-1}\sin(2n-1)x$$