Find the Fourier expansion of the function $f(x)=x^2$; $(-\pi<x<\pi)$. [98 高應大機械 7]

[解] $f(x)=\dfrac{a_0}{2}+\sum_{n=1}^{\infty}(a_n\cos nx+b_n\sin nx)$,

$$a_0=\frac{1}{\pi}\int_{-\pi}^{\pi}x^2dx=\frac{2\pi^2}{3},$$

$$a_n=\frac{1}{\pi}\int_{-\pi}^{\pi}x^2\cos nxdx=\frac{2}{n\pi}(x^2\sin nx\Big|_0^{\pi}-\int_0^{\pi}2x\sin nxdx)$$

$$=\frac{4}{n^2\pi}(x\cos nx\Big|_0^{\pi}-\int_0^{\pi}\cos nxdx)=\frac{4}{n^2\pi}[(-1)^n\pi]=(-1)^n\frac{4}{n^2},$$

$$b_n=0,$$

$$f(x)=\frac{\pi^2}{3}+4\sum_{n=1}^{\infty}(-1)^n\frac{\cos nx}{n^2},\ -\pi<x<\pi$$