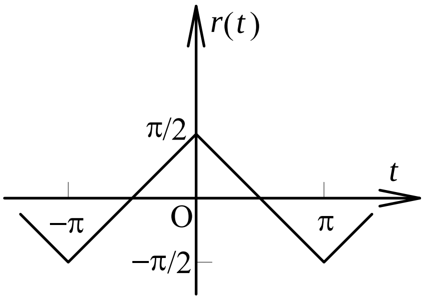
Find the Fourier series of the periodic function $r(t)$ of period $p=2\pi$, as shown below. [96 中央機械二 2]



[解]$r(t)=\begin{cases}-\dfrac{\pi}{2}+t, & -\pi<t<0\\ \dfrac{\pi}{2}-t, & 0<t<\pi\end{cases}\Rightarrow r(t)$為偶函數$\Rightarrow r(t)=\dfrac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos nt$

$$a_0=\frac{2}{\pi}\int_0^{\pi}r(t)dt=\frac{2}{\pi}\int_0^{\pi}(\frac{\pi}{2}-t)dt=\frac{2}{\pi}\cdot(\frac{\pi}{2}t-\frac{t^2}{2})\Big|_0^{\pi}=\frac{2}{\pi}\cdot(\frac{\pi^2}{2}-\frac{\pi^2}{2})=0$$

$$a_n=\frac{2}{\pi}\int_0^{\pi}r(t)\cos nt dt=\frac{2}{\pi}\int_0^{\pi}(\frac{\pi}{2}-t)\cos nt dt=\frac{2}{n\pi}[(\frac{\pi}{2}-t)\sin nt\Big|_0^{\pi}+\int_0^{\pi}\sin nt dt]$$

$$=\frac{2}{n\pi}(0-\frac{\cos nt}{n}\Big|_0^{\pi})=-\frac{2}{n^2\pi}(\cos n\pi-1)=-\frac{2}{n^2\pi}[(-1)^n-1]=\frac{4}{(2n-1)^2\pi}$$

$$r(t)=\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{\cos(2n-1)t}{(2n-1)^2}$$