

Please find the Fourier series of the periodic function:  $f(x) = x, |x| < T$ . [95中正機械2(b)]

[解]  $f(x)$  為奇函數，週期為  $2T \Rightarrow$  設  $f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{T}$

$$b_n = \frac{2}{T} \int_0^T f(x) \sin \frac{n\pi x}{T} dx = \frac{2}{T} \int_0^T x \sin \frac{n\pi x}{T} dx = -\frac{2}{n\pi} \left( x \cos \frac{n\pi x}{T} \Big|_0^T - \int_0^T \cos \frac{n\pi x}{T} dx \right)$$

$$= -\frac{2}{n\pi} \left( T \cos n\pi - \frac{T}{n\pi} \sin \frac{n\pi x}{T} \Big|_0^T \right) = -(-1)^n \frac{2T}{n\pi} = (-1)^{n+1} \frac{2T}{n\pi}$$

$$f(x) = \frac{2T}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin \frac{n\pi x}{T}$$

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