

Expand  $f(x) = \begin{cases} -1, & -\pi < x < 0 \\ 1, & 0 \leq x < \pi \end{cases}$  in a complex Fourier series. [93 中央機械 8(b)]

[解] 令  $f(x) = \sum_{n=-\infty}^{\infty} c_n e^{inx}, \quad -\pi < x < \pi$

$$c_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) e^{-inx} dx = \frac{1}{2\pi} \left[ \int_{-\pi}^0 -1 \cdot e^{-inx} dx + \int_0^{\pi} 1 \cdot e^{-inx} dx \right] = \frac{1}{2\pi} \left( \left. \frac{e^{-inx}}{-in} \right|_{-\pi}^0 + \left. \frac{e^{-inx}}{-in} \right|_0^{\pi} \right)$$

$$= \frac{1}{2\pi} \left( \frac{1 - e^{in\pi}}{-in} + \frac{e^{-in\pi} - 1}{-in} \right) = \frac{1}{2\pi} \left( \frac{2}{in} - \frac{e^{in\pi}}{in} - \frac{e^{-in\pi}}{in} \right)$$

$$= \frac{1}{2\pi} \left[ \frac{2}{in} - \frac{\cos n\pi + i \sin n\pi}{in} - \frac{\cos(-n\pi) + i \sin(-n\pi)}{in} \right]$$

$$= \frac{1}{2\pi} \left[ \frac{2}{in} - \frac{(-1)^n}{in} - \frac{(-1)^n}{in} \right] = \frac{1}{\pi} \left[ \frac{1 - (-1)^n}{in} \right] = \frac{2}{i\pi(2n-1)}$$

$$\therefore f(x) = \frac{2}{i\pi} \sum_{n=-\infty}^{\infty} \frac{1}{2n-1} e^{i(2n-1)x}, \quad -\pi < x < \pi$$