

Find the Fourier transform of e^{-ax^2} , where $a > 0$. [89 成大機械 5]

$$[\text{解}] \int_{-\infty}^{\infty} f(x) e^{-i\omega x} dx = \int_{-\infty}^{\infty} e^{-ax^2} e^{-i\omega x} dx = \int_{-\infty}^{\infty} e^{-a(x^2 + \frac{i\omega}{a}x)} dx = \int_{-\infty}^{\infty} e^{-a(x + \frac{i\omega}{2a})^2 - \frac{\omega^2}{4a}} dx = e^{-\frac{\omega^2}{4a}} \int_{-\infty}^{\infty} e^{-a(x + \frac{i\omega}{2a})^2} dx$$

令 $u = x + \frac{i\omega}{2a}$, 上式為

$$e^{-\frac{\omega^2}{4a}} \int_{-\infty}^{\infty} e^{-au^2} du = e^{-\frac{\omega^2}{4a}} \int_{-\infty}^{\infty} e^{-(\sqrt{a}u)^2} du = e^{-\frac{\omega^2}{4a}} \cdot \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} e^{-(\sqrt{a}u)^2} d(\sqrt{a}u) = e^{-\frac{\omega^2}{4a}} \cdot \sqrt{\frac{\pi}{a}}$$

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