

Let $f(x) = e^{-|x|}$, compute the complex Fourier integral of $f(x)$. Note that $f(x) = \begin{cases} e^{-x}, & x \geq 0 \\ e^x, & x < 0 \end{cases}$. [88 成

大機械 5]

[解] 令 $f(x) = \int_{-\infty}^{\infty} c(\omega) e^{i\omega x} d\omega$

$$c(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(x) e^{-i\omega x} dx = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-|x|} e^{-i\omega x} dx = \frac{1}{2\pi} \left(\int_{-\infty}^0 e^x e^{-i\omega x} dx + \int_0^{\infty} e^{-x} e^{-i\omega x} dx \right)$$

$$= \frac{1}{2\pi} \left(\int_{-\infty}^0 e^{(1-i\omega)x} dx + \int_0^{\infty} e^{-(1+i\omega)x} dx \right) = \frac{1}{2\pi} \left(\left. \frac{e^{(1-i\omega)x}}{1-i\omega} \right|_{-\infty}^0 - \left. \frac{e^{-(1+i\omega)x}}{1+i\omega} \right|_0^{\infty} \right)$$

$$= \frac{1}{2\pi} \left(\frac{1-0}{1-i\omega} - \frac{0-1}{1+i\omega} \right) = \frac{1}{2\pi} \cdot \frac{(1+i\omega) + (1-i\omega)}{(1-i\omega)(1+i\omega)} = \frac{1}{\pi(1+\omega^2)}$$

$$f(x) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{1}{1+\omega^2} e^{i\omega x} d\omega$$

以 $f(x) = \frac{1}{\pi} \int_{-u}^u \frac{1}{1+\omega^2} e^{i\omega x} d\omega$ 畫圖，如下

