

Show that the Fourier transformation of $e^{-\alpha t^2}$ is $\sqrt{\frac{\pi}{\alpha}} e^{-\frac{\omega^2}{4\alpha}}$. [106 台大機械 5(a)]

$$\begin{aligned} [\text{解}] \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt &= \int_{-\infty}^{\infty} e^{-\alpha t^2} e^{-i\omega t} dt = \int_{-\infty}^{\infty} e^{-\alpha(t^2 + \frac{i\omega}{\alpha}t)} dx = \int_{-\infty}^{\infty} e^{-\alpha(t + \frac{i\omega}{2\alpha})^2 - \frac{\omega^2}{4\alpha}} dt \\ &= e^{-\frac{\omega^2}{4\alpha}} \int_{-\infty}^{\infty} e^{-\alpha(t + \frac{i\omega}{2\alpha})^2} dt \end{aligned}$$

令 $u = t + \frac{i\omega}{2\alpha}$, 上式為

$$e^{-\frac{\omega^2}{4\alpha}} \int_{-\infty}^{\infty} e^{-\alpha u^2} du = e^{-\frac{\omega^2}{4\alpha}} \int_{-\infty}^{\infty} e^{-(\sqrt{\alpha}u)^2} du = e^{-\frac{\omega^2}{4\alpha}} \cdot \frac{1}{\sqrt{\alpha}} \int_{-\infty}^{\infty} e^{-(\sqrt{\alpha}u)^2} d(\sqrt{\alpha}u) = e^{-\frac{\omega^2}{4\alpha}} \cdot \sqrt{\frac{\pi}{\alpha}}$$

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